Global Shift Analysis of Dynamic Density Functions by Structured ΣΠ-Networks

Helmut Glünder

Institut für Medizinische Psychologie, Ludwig-Maximilians-Universität
Gœthestraße 31, D-8000 München 2, Germany

A theorem is proven which serves as the mathematical basis for the proposed global shift-vector extraction. This theorem states the identity of the difference vector between the centers of gravity (centroids) of two arbitrary nD density functions, with the centroid vector of their cross-correlation function. Consequently, the centroid of the cross-correlation function of consecutive manifestations of an arbitrarily transforming density function indicates its incremental shift vector. Advantages of this approach for implementations in massively parallel computing structures as well as applications, such as visual velocity estimation of nonrigid objects, are discussed.

Introduction

The extraction of spatially global and temporally incremental changes from variations of a nonrigid density function or activity distribution is requisite for the visual analysis of object motion in 3D space and, more generally, for the evaluation of distributed dynamic data representations that serve for fault-tolerant computing, etc. The sometimes rather difficult task of analysing well-defined geometric transformations of spatial density functions is essentially complicated by the demand for treating arbitrary changes. The here considered global analysis of the purely translatory portion of such general transformations of a density function must be based on characteristic points, most naturally the center of gravity (centroid). As it is known, the centroid location of a density function (non-negative!) is determined according to Eq. (2a) below. (The centroid of a real-valued function exists if its integral value is different from zero). Obviously, the desired global shift vector which relates the centroid positions $\vec{R}_{t-\tau_0}$ and $\vec{R}_t$ of successive (time interval τ_0) manifestations $a_{t-\tau_0}(\vec{r},t-\tau_0)$ and $a_t(\vec{r},t)$ of a changing density function is given by the difference vector

$$\Delta\vec{R}_t = \vec{R}_t - \vec{R}_{t-\tau_0} \,. \tag{1}$$

In contrast to this straightforward solution, a method for the extraction of the global shift vector $\Delta\vec{R}_t$ is proposed for which the centroid locations of the two temporally separated functions $a_{t-\tau_0}(\vec{r},t-\tau_0)$ and $a_t(\vec{r},t)$ are not required. This approach is based on the cross-correlation function of these two functions and the method is especially advantageous if massively parallel computing structures are considered and if the shift increments and the spatial extent of the density function are both small compared to the space to which the function's position is confined.

The following section presents the mathematical core of the method. Next, an appropriate two-stage computing structure is introduced, and finally applications are discussed.

The Theorem

The ith component X_{ai} of the centroid vector $\vec{R}_a = (X_{a1} \ldots X_{ai} \ldots X_{an})^{\mathrm{T}}$ of a real-valued nD function $a(\vec{r})$ with $\vec{r} = (x_1 \ldots x_i \ldots x_n)^{\mathrm{T}}$ is defined by the nD integral

$$X_{ai} = \frac{1}{A} \int a(\vec{r}) \cdot x_i \, \mathrm{d}\vec{r} \qquad \text{with} \quad A = \int a(\vec{r}) \, \mathrm{d}\vec{r} \tag{2a}$$

or, using the projection of $a(\vec{r})$ along all $(n-1)$ coordinates $x_j \neq x_i$

$$a_i(x_i) = \int a(\vec{r}) \, \mathrm{d}\vec{\vec{r}} \qquad \text{with} \quad \mathrm{d}\vec{\vec{r}} = (\mathrm{d}x_1 \ldots \mathrm{d}x_i = 0 \ldots \mathrm{d}x_n)^{\mathrm{T}}, \tag{3}$$

by the 1D integral

$$X_{ai} = \frac{1}{A} \int a_i(x_i) \cdot x_i \, \mathrm{d}x_i \, . \tag{2b}$$

By analogy, the ith component Ξ_{ki} of the centroid vector of the cross-correlation function

$$k_{ba}(\vec{\rho}) = \int b(\vec{r}) \cdot a(\vec{r} - \vec{\rho}) \, \mathrm{d}\vec{r} \qquad \text{with} \quad \vec{\rho} = (\xi_1 \ldots \xi_i \ldots \xi_n)^{\mathrm{T}} \tag{4}$$

of the functions $b(\vec{r})$ and $a(\vec{r})$ is given by

$$\Xi_{ki} = \frac{1}{K} \int k_{bai}(\xi_i) \cdot \xi_i \, \mathrm{d}\xi_i \qquad \text{with} \quad K = B \cdot A \tag{5a}$$

or, in a detailed and more convenient form:

$$\Xi_{ki} = \frac{1}{B} \int b_i(x_i) \Big[\frac{1}{A} \int a_i(x_i - \xi_i) \cdot \xi_i \, \mathrm{d}\xi_i \Big] \mathrm{d}x_i \, . \tag{5b}$$

The inner integral of Eq. (5b) represents the ith component

$$\Xi_{ai}(x_i) = \frac{1}{A} \int a_i(x_i - \xi_i) \cdot \xi_i \, \mathrm{d}\xi_i = x_i - X_{ai} \tag{6}$$

of the centroid vector of function $a_i(x_i - \xi_i)$ or, in other words, of the centroid vector of function $a_i(-\xi_i)$ shifted according to x_i. Inserting the rightmost expression of Eq. (6) into Eq. (5b) yields

$$\Xi_{ki} = \frac{1}{B} \int b_i(x_i) \cdot x_i \, \mathrm{d}x_i - \frac{X_{ai}}{B} \int b_i(x_i) \, \mathrm{d}x_i = X_{bi} - X_{ai} = \Delta X_{bai} \, , \tag{5c}$$

and reveals the claimed identity of the components Ξ_{ki} of the centroid vector of the cross-correlation function $k_{ba}(\vec{\rho})$ computed from the two functions $b(\vec{r})$ and $a(\vec{r})$, with the differences ΔX_{bai} of the corresponding components X_{bi} and X_{ai} of the centroid vectors of these two functions (cf. Eq. (1)). Another proof can be based on the convolution-theorem used in statistics which deals with means of probability densities (see [1], p.266).

Network Structure

According to the initial problem, the first processing stage must compute the time-dependent cross-correlation function

$$k_t(\vec{\rho}, t) = \int a_t(\vec{r},t) \cdot a_{t-\tau_0}(\vec{r} - \vec{\rho}, t - \tau_0)\, \mathrm{d}\vec{r}\,. \tag{7}$$

It was shown in previous articles [2] and [3], how this can be achieved by a structured ΣΠ-network [4] of polynomial order 2 with constant delays τ_0 applied to every second input. Such a network consists of many multiplying subunits of the basic Hassenstein/Reichardt-type [5] sketched in Fig. 1, each receiving its input signals from one pair of points in the considered input space. The output signals of all possible subunits are separately summed – according to the difference vectors $\vec{\rho}$ of the points – by so-called ΣΠ-units. At every time t, the resulting ensemble of sums represents the desired correlation function of two successive manifestations of whatever dynamic event in the input space.

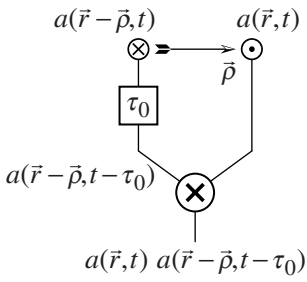

Figure 1. Subunit for the detection of spatio-temporal signal relations

For realistic problems, the number of ΣΠ-units will be prohibitively large, at least for technical implementations. However, if the individual shifts of all points of the density function are confined to a reasonably low value $\vec{\rho}_{\max}$, e.g. by choosing a sufficiently short temporal interval τ_0, then, the number of ΣΠ-units is dramatically reduced. Under such realistic restrictions, the resulting correlation function will have an enormously lesser extent than the input space. Furthermore, a modular network design (local correlations) is possible if the following minimum conditions are met. In order to permit the calculation of correct global correlation coefficients by simple summation of the local results, the smallest spatial module must have an extent of $2|\vec{\rho}|$ and neighbouring modules must overlap at least by $|\vec{\rho}|$ (cf. [6]).

In a second stage the centroid location of the correlation function can be computed by feeding its values via appropriately weighted interconnections to summation nodes. The values at these nodes, divided by the spatial integral value of the correlation function, are the desired coordinates of the centroid (cf. Eq. (2a)).

The described two-stage network delivers the continuous time-course of the incremental (τ_0) global shift vector of an arbitrary dynamic density function in the input space. At the output of the first stage the shift vector is represented as an activity distribution (population coding). This representation is uniquely related to the origin of the coordinate sys-

tem. In a second stage, the numerical value of the components of the shift vector can be computed by noise-insensitive integral operations, namely by the centroid extraction of this origin-related activity distribution. The computational accuracy is expected to be higher than for the straightforward method, because subtractions of large (coordinate) values do not occur. Besides the extraction of the global shift, local correlation can serve for other purposes as well, e.g. for the estimation of local deformations of the density function.

Applications

Surely the most prominent application of the described method is in the field of visual motion analysis, especially for tracking tasks, where real-time operation is of utmost importance. Obviously, the shift vector – after its division by the time delay τ_0 – turns into an estimate of the true instantaneous velocity vector. It was shown elsewhere [2] that, in the case of centroid-based evaluations of the cross-correlation function $k_t(\vec{\rho}, t)$, comparable estimates can be achieved by applying realizable low-pass filters (impulse response $h(t)$) instead of idealized delay-filters to the subunits. In these cases, the estimated time-course of the velocity is a low-pass filtered (impulse response $g(t)$) version of the time-course of the true velocity, with

$$g(t) = \frac{1}{\tau_h}\left[1\Big|_{t\geq 0} - \frac{1}{H}\int_0^t h(t)\,\mathrm{d}t\right], \quad \tau_h = \frac{1}{H}\int h(t)\cdot t\,\mathrm{d}t \text{ and } H = \int h(t)\,\mathrm{d}t\,. \tag{8}$$

Although these properties were originally derived for rigid objects that translate in the analyser plane, they still hold true for the here discussed general case. In short, it is possible to estimate – with known accuracy – the time-course of the centroid velocity of the 2D central projection of an (non-rigid) object that arbitrarily moves in 3D space in front of an unstructured background. More elaboration – especially of the local analyser concept – is needed in order to be able to deal with structured backgrounds and with several independently moving objects.

A more general application, not restricted to the analysis of shifts in two dimensions, is the evaluation of the dynamics of distributed data representations in network structures. Although networks are confined to three dimensions, higher-dimensional data can be represented in 3D or even 2D networks. Shifts of activity in such data-spaces, as Abeles ([7], p.74) conjectures them to occur in cortical circuits, can also be analysed by the computational means described in the previous section, provided the adequate interconnection structure is implemented. Advantages of such highly redundant representations are their fault-tolerance as well as low demands concerning range and resolution of the values in the representation. There is good evidence that such principles are used by biological neural systems (see Abeles [7] and for a review Sejnowski [8]). Whether these ideas will be converted into technical applications is up to the future.

Acknowledgements

The author is supported by the "Volkswagen-Stiftung" under the grant I/65 914; he is grateful to E. Pöppel for encouragement as well as to R. Bamler for an essential hint.

References

[1] A. Papoulis. *Probability, Random Variables, and Stochastic Processes.* McGraw-Hill, New York, 1965.

[2] H. Glünder. Correlative velocity estimation: visual motion analysis, independent of object form, in arrays of velocity-tuned bilocal detectors. *J. Opt. Soc. Am. A* **7**: 255-263, 1990.

[3] H. Glünder. ΣΠ-networks for motion and invariant form analyses. In: R. Eckmiller, G. Hartmann and G. Hauske (ed.) *Parallel Processing in Neural Systems and Computers.* Elsevier, Amsterdam, pp. 357-360, 1990.

[4] D.E. Rumelhart, G.E. Hinton and J.L. McClelland. A general framework for parallel distributed processing. In: D.E.Rumelhart and J.L. McClelland (ed.) *Parallel Distributed Processing 1.* MIT Press, Cambridge/MA, pp. 45-76, 1986.

[5] B. Hassenstein and W. Reichardt. Systemtheoretische Analyse der Zeit-, Reihenfolgen- und Vorzeichenauswertung bei der Bewegungsperzeption des Rüsselkäfers Chlorophanus. *Z. Naturforschg.* **11b**: 513-524, 1956.

[6] H. Glünder. A dualistic view of motion and invariant shape analysis. In: J.C. Simon (ed.) *From Pixels to Features.* North-Holland, Amsterdam, pp. 323-332, 1989.

[7] M. Abeles. *Local Cortical Circuits.* Springer, Berlin, 1982.

[8] T.J. Sejnowski. Neural populations revealed. *Nature* **332**: 308, 1988.

Bernd Radig (Hrsg.)

Mustererkennung 1991

13. DAGM-Symposium
München, 9.-11. Oktober 1991

Proceedings

Unter Mitarbeit von Karlhorst Klotz

Springer-Verlag
Berlin Heidelberg New York London Paris
Tokyo Hong Kong Barcelona Budapest