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INVARIANT DESCRIPTION OF PICTORIAL PATTERNS VIA GENERALIZED AUTO-CORRELATION FUNCTIONS

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Abstract. A systematic approach to geometrically invariant pattern description is proposed. It is based on the definition of transformation invariants. The generalized auto-correlation function is introduced as a signal representation from which such invariant descriptors can be derived. Descriptors remaining invariant under all similarity transformations are briefly discussed.

Introduction

In contrast to coding (redundancy removal), pattern description implies a considerable loss of information (irrelevance) and thus in general does not permit unequivocal signal reconstruction. Pattern descriptions are essentially determined by specific tasks and pattern repertoires. Especially pictorial patterns represent enormous amounts of information and thus allow for large varieties of descriptions. This manifold, however, is reduced by quite a pragmatic demand: the invariance of pattern descriptions under geometric transformations. Although it is intuitively clear that increased invariance causes decreased richness of description, only few systematic and practical approaches to the *principal limits* of invariant pattern description are known. The following considerations are based on the definition of transformation invariants and thus are independent from both the purpose of descriptions (except for the claim for invariance itself) and the patterns to be described. Consequently, the resulting descriptions cannot be based on any kind of prototype detection, e.g., on *pattern elements*. In fact, they express general but nonetheless obvious *pattern properties*; mainly various types of symmetry (cf. Radig and Schlieder 1984), i.e., categories outside the reach of template matching techniques, including integral transformations.

Limits of invariant pattern description

The limits of invariant pattern description are given by the invariants of geometric transformations. These are those properties of pattern functions that remain unaltered under certain transformations; for example:

GROUP OF TRANSFORMATIONS	INVARIANTS
• projective	cross-ratio (length double-ratio)
• affine	+ area ratio, length ratio, parallelism
• similarity	+ angle
• congruence	+ area, length

Therefore, invariants represent the categories of invariant pattern description. But how can they be dealt with in practice? – Optimal invariant pattern description, with respect to certain transformations, means to specify the degree to which invariant properties are present in a pattern. Testing this for a single type of transformation is easy: The pattern is transformed and, for every value of the transformation parameter, the distance to the untransformed pattern is determined according to a suitable measure. The resulting comparison function, by definition is invariant under the applied transformation (inherent invariance). In general, this does not hold for several types of transformations, e.g., groups. In these cases the multiple-invariant intersection of the sets of invariants must be determined.

Generalized auto-correlation function

A mathematical formulation of the aforementioned principle is introduced by the generalized auto-correlation function (ACF) (cf. Glünder et al. 1984, and Strube 1985).

$$c(\mathfrak{z}_1 \ldots \mathfrak{z}_m) = \int \bigwedge_{i=1}^{m} [\, b(\mathcal{T}_i\, \boldsymbol{x})\,]\, d\boldsymbol{x}, \qquad \text{for} \quad b(\boldsymbol{x}) \geq 0 \tag{1}$$

with $\bigwedge_{i=1}^{m} [\ldots]$, symbolizing a mathematical operation on m terms; e.g., multiplication: $\prod_{i=1}^{m} [\ldots]$;

and $\mathcal{T}_i = \mathcal{T}(\mathfrak{z}_i)$, indicating geometric transformations, characterized by the ν-dimensional vector $\mathfrak{z}_i = (z_{i1} \ldots z_{i\nu})^T$, and acting on the n-dimensional argument $\boldsymbol{x}$ of a pattern function $b(\boldsymbol{x})$;

According to the last section, the extraction of invariant properties from pictorial patterns (n=2) is based on comparisons between a pattern and its transformed versions (m=2). Furthermore, the following investigations are confined to multiplicative comparisons, and to the group of similarity transformations (ν=5). Therefore, equation (1) can be written in the following way

$$c_{sim}(\xi,\eta,s,\psi,\sigma) = \iint b(x,y) \cdot b\{[r/s \cdot \cos(\varphi+\psi)]+\xi,\ [\sigma \cdot r/s \cdot \sin(\varphi+\psi)]+\eta\}\ dxdy, \tag{2}$$

with $r = \sqrt{x^2 + y^2}$ and $\varphi = \arctan(y/x)$; where $\sigma = -1$ leads to reflections at the x-axis.

The thus defined (second order) similarity-ACF *contains* invariant information about the shape (in the mathematical sense) of patterns. Invariance can be expected with regard to all similarity transformations; i.e., to changes of size (s), sense (σ), as well as shift (ξ,η) and rotational (ψ) position of a pattern.

Similarity-invariant descriptors

The complete similarity-ACF is *not invariant at all.* However, three subspaces are invariant under the group of similarity transformations if one gets rid of the constraints imposed by the fixed points of the rotations, scale changes, and reflections. The most pronounced (primary) descriptors are obtained via maximum detection with respect to the limiting parameters.

$$C_s(s) = \max_{\xi,\eta} \{c_{sim}(\xi,\eta,s,\psi=0^\circ,\sigma=+1)\} \qquad \text{with } 0 < s \leq 1, \text{ or } \quad 1 \leq s < \infty$$

$$C_\psi(\psi) = \max_{\xi,\eta} \{c_{sim}(\xi,\eta,s=1,\psi,\sigma=+1)\} \qquad \text{with } 0^\circ \leq \psi \leq 180^\circ$$

$$C_\sigma(\sigma) = \max_{\xi,\eta,\psi} \{c_{sim}(\xi,\eta,s=1,\psi,\sigma)\} \qquad \text{with } \sigma = \pm 1 \quad \text{and } 0^\circ \leq \psi < 360^\circ$$

The descriptor C_s indicates the maximum degree of size-similarity of a pattern as a function of scale. Hence, patterns consisting of straight lines that join or intersect at a common point result in C_s=const. Contrarily, any fraction of a circle line leads to a sudden decrease of this function for $s \neq 1$. The descriptor C_ψ reveals the maximum angular bindings of a pattern; thus, a perfect circle is the only pattern with C_ψ=const. Patterns of at least one perfect axial symmetry are identified by $C_\sigma(-1)=C_\sigma(+1)$. In all the other cases the coefficient $C_\sigma(-1)$ expresses the maximum degree of glide-reflection symmetry.
Compound descriptors show restricted invariance, for example:

$$C_1(\psi,\sigma) = \max_{\xi,\eta} \{c_{sim}(\xi,\eta,s=1,\psi,\sigma)\} \qquad \text{with } 0^\circ \leq \psi < 360^\circ \qquad \text{(rotation variant)}$$

$$C_2(s,\psi) = \max_{\xi,\eta} \{c_{sim}(\xi,\eta,s,\psi,\sigma=+1)\} \qquad \text{with } 0 < s \leq 1 \text{ and } 0^\circ \leq \psi < 360^\circ, \text{ or } 0 < s < \infty \text{ and } 0^\circ \leq \psi < 180^\circ. \qquad \text{(sense variant)}$$

Each value of all these descriptor functions reflects the maximum degree of pattern binding for the optimal shift position(s). However, no information is obtained about its relation to values for other positions. Consequentely, more of the invariant information is revealed, if maximum detection is accompanied by other, e.g., statistical methods, such as analysis of variance, etc.

Related descriptors

There are no *straightforward* descriptors invariantly indicating shift congruence of a pattern. Doyle (1962) showed, however, that such descriptors can be derived from the translational (classical) ACF $c_t(\xi,\eta)$. For this purpose, pattern b in equation (2) is replaced by its ACF c_t, which is a centred and central-symmetric function. Thus, the shift transformations are obsolete and the ranges of the rotation angles are halved. The three resulting (secondary) descriptors express inner bindings of the translational ACF, with respect to size, angle and sense. Although seemingly similar to the primary descriptors, they are of significantly reduced descriptive power and less obvious, but well-suited to supplement the primary descriptors.
Another quite important approach uses the fixed point function f_2, instead of pattern b in equation (2).

$$f_2(x_f,y_f) = \iint c_{f2}'(x_f,y_f,s,\psi)\, ds d\psi$$

The integrand c_{f2}' is a binary function which is derived from the similarity-ACF. It contains the 'optimal' fixed point coordinates (x_f,y_f), i.e., those shift positions that lead to the maxima of descriptor C_2. The resulting descriptors express highly abstract pattern properties, such as compactness, regularity, etc. Besides this, the fixed point that appears most frequently can serve for a final pattern-coherent description.

Concluding remarks

The generalized second order ACF represents a reasonable basis and a tool for the creation of a family of invariant pattern descriptors. For the case of similarity-invariance, indications are given for a systematic approach to the limits of pattern description, as defined by the group-invariants. There are two fundamentally different strategies to compute the generalized ACF: the 'explicit' one, where geometric transformations are actually performed, and the 'implicit' one, which is based on generalized dipole moments. The examples shown on the next page were computed, following an explicit technique, by correlating the pattern with its discretely transformed versions. Correlation and geometric transformations were performed by a new kind of incoherent-optical analog-correlator, maximum detection by digital evaluation. The implicit computation, based on comparisons between every two points of suitably chosen pairs of pixels, is advantageous for serial processing (cf. Glünder and Kramer 1986) but also for the implementation on special purpose parallel computing networks (Glünder 1986).

References

Doyle W (1962) Operations useful for similarity-invariant pattern recognition. J ACM **9**:259-267

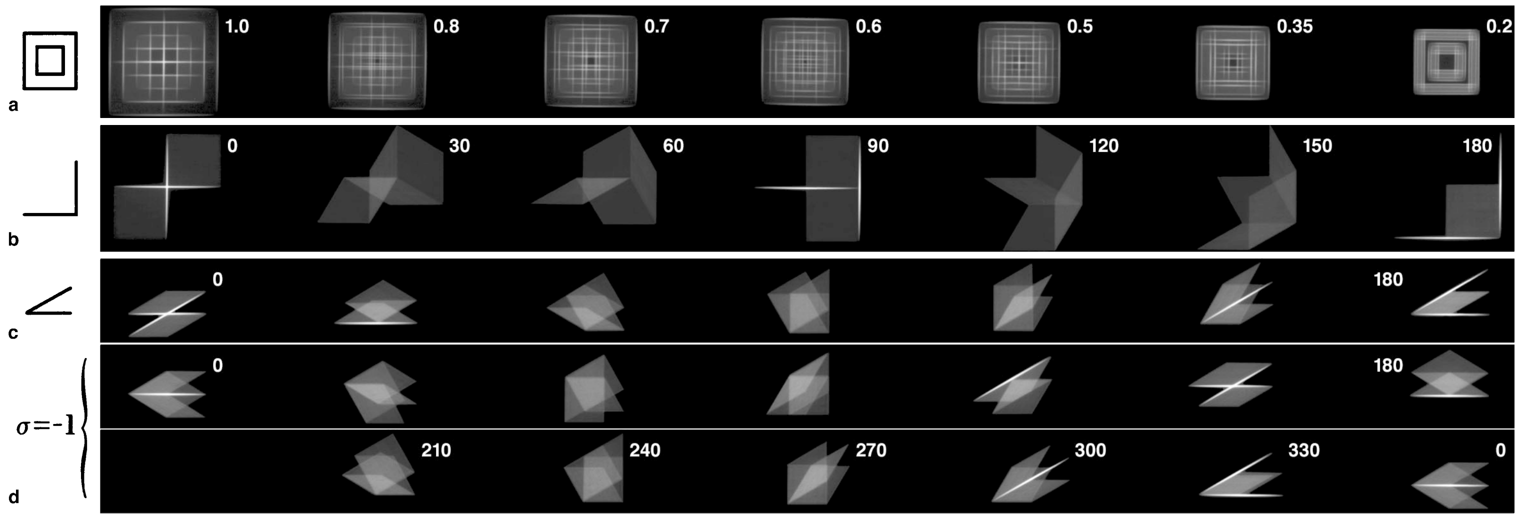
Glünder H (1986) Neural computation of inner geometric pattern relations. Biol Cybern **55**:239-151

Glünder H, Kramer T (1986) Description of planar patterns by invariant features – an attempt towards the explanation of visual pattern recognition. In: Guiho G (ed) Proc of 8th ICPR. IEEE Comp Soc Press, Washington DC, pp 1090-1093

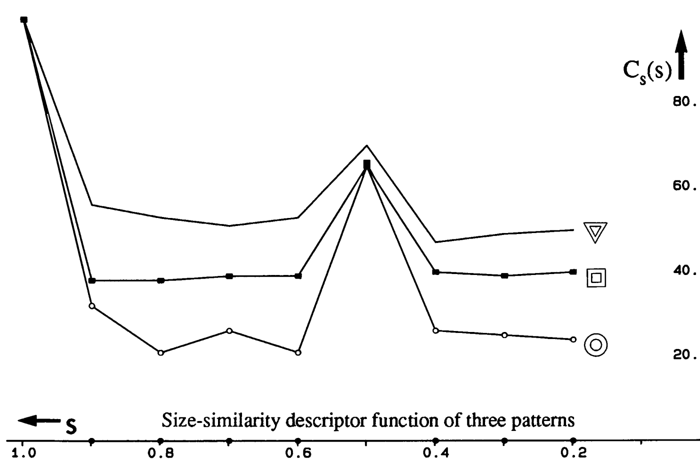
Glünder H, Gerhard A, Platzer H, Hofer-Alfeis J (1984) A geometrical-transformation-invariant pattern recognition concept incorporating elementary properties of neural circuits. In: Wein M (ed) Proceedings of the 7th International Conference on Pattern Recognition (ICPR). IEEE Comp Soc Press, Washington DC, pp 1376-1379

Radig B, Schlieder C (1984) RS-automorphisms and symmetrical objects. In: Wein M (ed) Proceedings of the 7th International Conference on Pattern Recognition (ICPR). IEEE Comp Soc Press, Washington DC, pp 1138-1140

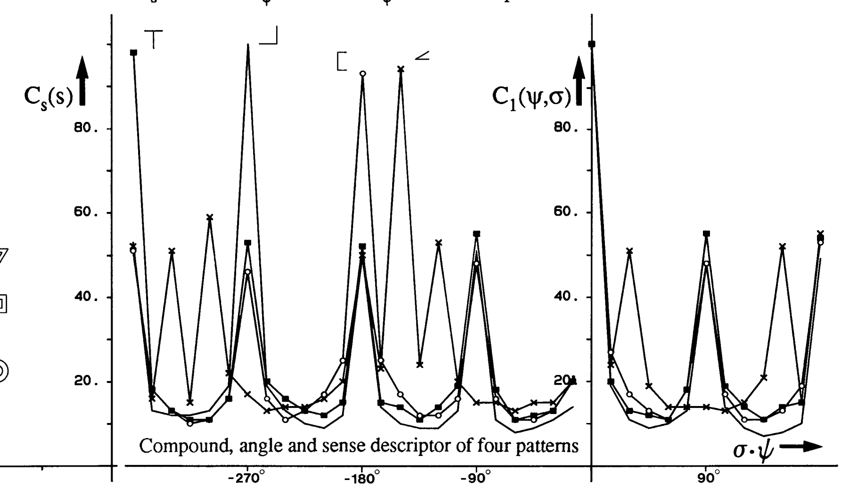
Strube HW (1985) A generalization of correlation functions and the Wiener-Khinchin theorem. Signal Process **8**:63-74



Three patterns and parts of various generalized ACFs; (a) $c_s(\xi,\eta,s)$; (b) $c_\psi(\xi,\eta,\psi)$; (c) $c_\psi(\xi,\eta,\psi)$; (d) $c_1(\xi,\eta,\sigma\cdot\psi)$;



Size-similarity descriptor function of three patterns



Compound, angle and sense descriptor of four patterns