

INVARIANT DESCRIPTION OF PICTORIAL PATTERNS VIA GENERALIZED AUTO-CORRELATION FUNCTIONS

Helmut Glünder

Lehrstuhl für Nachrichtentechnik, Technische Universität München
Arcisstr. 21, D-8000 München 2, FRG

Abstract. A systematic approach to geometrically invariant pattern description is proposed. It is based on the definition of transformation invariants. The generalized auto-correlation function is introduced as a signal representation from which such invariant descriptors can be derived. Descriptors remaining invariant under all similarity transformations are briefly discussed.

Introduction

In contrast to coding (redundancy removal), pattern description implies a considerable loss of information (irrelevance). Thus, it does not generally permit of the unequivocal signal reconstruction. Pattern descriptions are essentially determined by specific tasks and pattern repertoires. Especially pictorial patterns commonly contain a lot of information, hence they can be described in very many ways. The variety however, becomes considerably restricted if the descriptions are to be invariant under geometric transformations. Although it is intuitively clear that increased invariance causes decreased richness of description, only few systematic and practicable approaches to the principal limits of invariant pattern description are known. The following considerations are based on the definition of transformation invariants. They neither depend on the purpose of the descriptions (except for the claim for invariance itself), nor on the patterns to be described. Consequently, the resulting descriptions are not based on any kind of pattern decomposition or prototype detection. In fact, they express inherent but nonetheless obvious pattern properties that turn out to be general symmetries (cf. Radig and Schlieder 1984), i.e. they belong to categories typically outside the reach of template matching techniques, integral transformations included.

Limits of invariant pattern description

The limits of invariant pattern description are given by the invariants of geometric transformations. They are those properties of pattern functions that remain unaltered under defined transformations, for example:

Transformation Group
projective
affine
similarity
congruence

Group Invariants
cross-ratio (length double-ratio)
+ area ratio, length ratio, parallelism
+ angle
+ area, length

Therefore, invariants represent the categories of invariant pattern description. But how can they be dealt with in practice? – Optimum invariant pattern description with respect to geometric transformations means to specify the degree to which invariant properties are present in a pattern. This can be easily accomplished for a single type of transformation: The pattern is transformed and the distance or likeness to the original is determined according to a suitable measure and as a function of the transformation parameters. By definition, the resulting comparison function is invariant under the applied transformation (inherent invariance). In general, this does not hold for composite transformations. The then required group invariants can be obtained as multiple-invariant intersections of sets of elementary invariants.

Generalized auto-correlation function

A mathematical formulation of the aforementioned principle is introduced by the generalized auto-correlation function (ACF) (Glünder *et al.* 1984, Strube 1985)

$$c(\bar{z}_1 \dots \bar{z}_m) = \int \bigwedge_{i=1}^m b(\mathcal{T}_i\{\bar{x}\}) d\bar{x} \quad \text{for real-valued patterns } b(\bar{x}), \quad (1)$$

where $\bigwedge_{i=1}^m \dots$ denotes a conjunctive operation on m terms, e.g. the product $\prod_{i=1}^m \dots$, and $\mathcal{T}_i = \mathcal{T}(\bar{z}_i)$ indicates geometric transformations with parameter vectors $\bar{z}_i = (z_{i1} \dots z_{iv})^T$ that are applied to the n -dimensional function $b(\bar{x})$.

According to the previous section, the invariants of a pictorial pattern ($n = 2$) result from comparing it to its geometrically transformed versions ($m = 2$). In what follows, the group of similarity transformations ($v = 5$) is considered and product comparisons are applied. Consequently, definition (1) can be written as

$$c_{\text{sim}}(\xi, \eta, s, \psi, \sigma) = \iint b(x, y) \cdot b\left(\frac{r}{s} \cos[\varphi - \psi] - \xi, \sigma \frac{r}{s} \sin[\varphi - \psi] - \eta\right) dx dy, \quad (2)$$

with $r = \sqrt{x^2 + y^2}$ and $\varphi = \arctan(y/x) + \pi \cdot \min\{\text{sgn}(x), 0\}$. (Although slightly less general than separate scale factors $s_x = s$ and $s_y = \sigma \cdot s$, more convenient formulations result from $s > 0$ and $\sigma = \pm 1$.) The such defined (second order) similarity-ACF contains information about a pattern's shape (in the mathematical sense). The group invariant subset of the descriptors is invariant under changes of size (s), sense (σ), and the translational (ξ, η) and rotational (ψ) positions of the pattern.

Similarity-invariant descriptors

In general, the similarity-ACF *per se* is not invariant – but at least three of its subspaces are. They are obtained by getting rid of constraints imposed by the fixed points of rotation, scale change, and reflection. Pronounced (primary) descriptors result from maximum detection:

$$\begin{aligned} C_s(s) &= \max_{\xi, \eta} \{c_{\text{sim}}(\xi, \eta, s, \psi = 0, \sigma = +1)\} & \text{with } 0 < s \leq 1, \text{ or } 1 \leq s < \infty \\ C_\psi(\psi) &= \max_{\xi, \eta} \{c_{\text{sim}}(\xi, \eta, s = 1, \psi, \sigma = +1)\} & \text{with } 0 \leq \psi < \pi \\ C_\sigma(\sigma) &= \max_{\xi, \eta, \psi} \{c_{\text{sim}}(\xi, \eta, s = 1, \psi, \sigma)\} & \text{with } 0 \leq \psi < 2\pi \text{ and } \sigma = \pm 1 \end{aligned}$$

The descriptor C_s indicates a pattern's maximum degree of similarity as a function of scale. Hence, self-similar patterns, such as straight lines that join or intersect at a common point, result in $C_s = \text{const.}$, whereas curved lines lead to a sudden decrease for $s \neq 1$. The descriptor C_ψ reveals the maximum angular pattern bindings. Consequently, perfect circles are the only patterns resulting in $C_\psi = \text{const.}$ Patterns showing at least one perfect axial symmetry are identified by $C_\sigma(-1) = C_\sigma(+1)$; otherwise, coefficient $C_\sigma(-1)$ expresses their maximum degree of glide-reflection symmetry.

In contrast, compound descriptors show restricted invariance. For example

$$\begin{aligned} C_1(\psi, \sigma) &= \max_{\xi, \eta} \{c_{\text{sim}}(\xi, \eta, s = 1, \psi, \sigma)\} & \text{with } 0 \leq \psi < 2\pi \text{ and } \sigma = \pm 1 & \text{ is rotation variant, and} \\ C_2(s, \psi) &= \max_{\xi, \eta} \{c_{\text{sim}}(\xi, \eta, s, \psi, \sigma = +1)\} & \text{with } 0 < s \leq 1 \text{ and } 0 \leq \psi < 2\pi, \\ & & \text{or } 1 \leq s < \infty \text{ and } 0 \leq \psi < \pi & \text{ is sense variant.} \end{aligned}$$

Any single value of the descriptor functions reflects the maximum degree of inner pattern binding with respect to the optimum translational position(s) but nothing is revealed about its relation to values resulting from different positions. Consequently, further invariant information can be obtained, if maximum detection is accompanied by other, e.g. statistical methods, such as analysis of variance, etc.

Related descriptors

There are no straightforward similarity-invariant descriptors that indicate a pattern's translatory congruences. However, Doyle (1962) showed that related features can be derived from the translational (classic) ACF $c_t(\xi, \eta)$. For this purpose, pattern $b(x, y)$ in equation (2) is replaced by its ACF c_t which is a centered function of central symmetry. Thus, translations are irrelevant and the ranges of the rotation angles are halved. The three resulting (secondary) descriptors express inner bindings of the translational ACF with respect to size, angle and sense. Although seemingly similar to the primary descriptors, they are of significantly reduced descriptive power and less obvious, but they nicely supplement the primary descriptors.

For another approach, pattern $b(x, y)$ in equation (2) is replaced by the fixed point function

$$f_2(x_f, y_f) = \iint \tilde{c}_{f2}(x_f, y_f, s, \psi) ds d\psi,$$

where the binary function \tilde{c}_{f2} is derived from the similarity-ACF, with x_f, y_f being the optimum fixed point coordinates underlying descriptor C_2 . The resulting (secondary) descriptors express fairly abstract pattern properties, such as compactness and regularity. Finally and as an aside: The most frequently appearing fixed point can serve as the center of a pattern-coherent description.

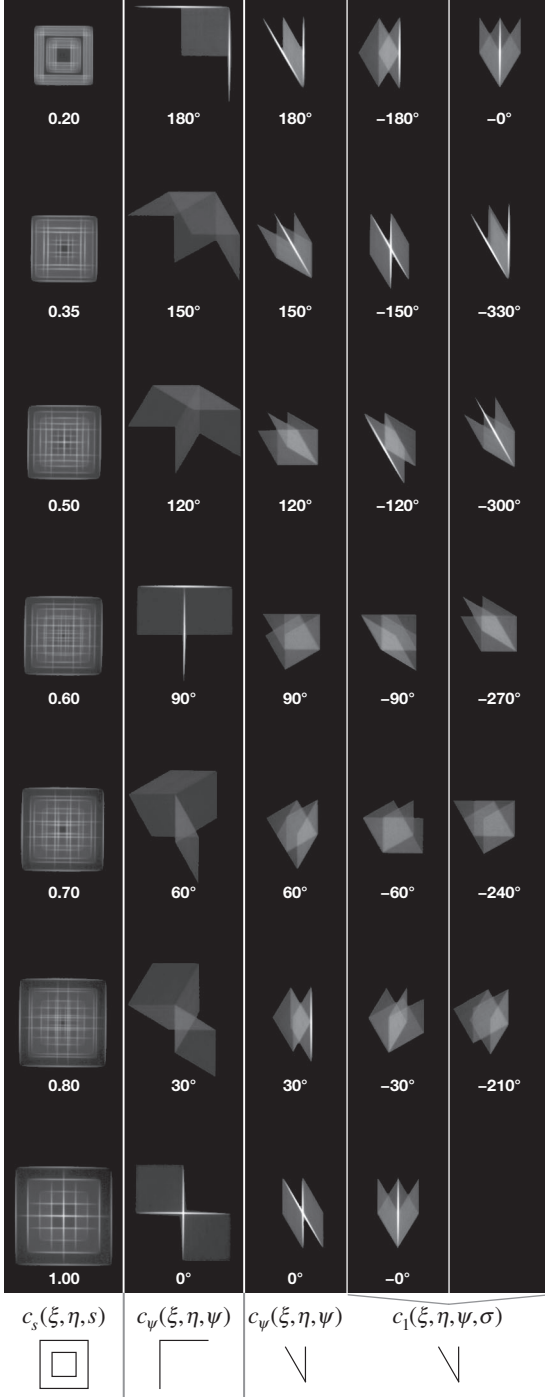
Concluding remarks

The generalized second order ACF was shown to provide a reasonable basis for the extraction of a family of invariant pattern descriptors. Furthermore, indications were provided of how to approach the limits of similarity-invariant pattern descriptions, as they are defined by the group invariants.

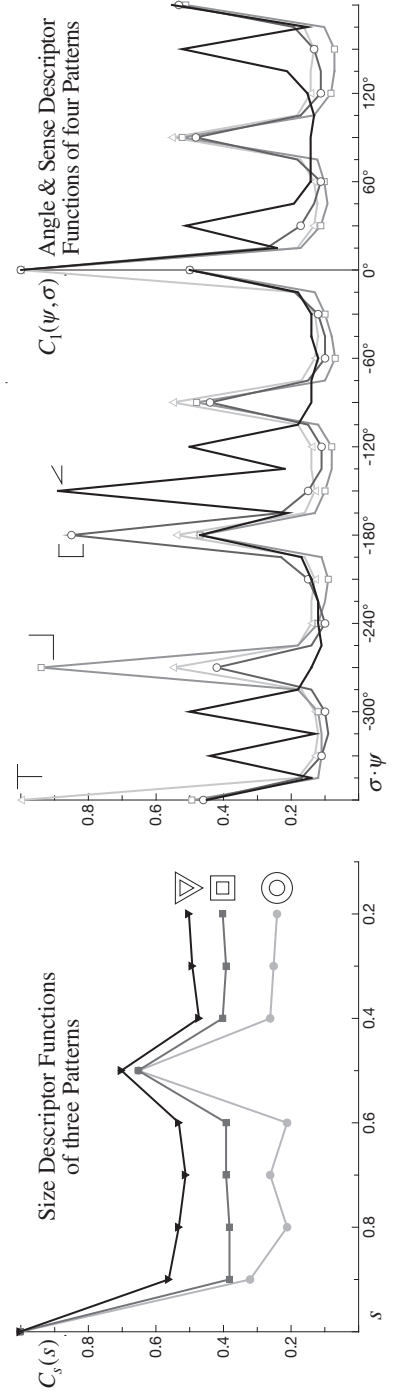
The generalized ACF can be obtained from two procedures: the "explicit" one, with actually performed geometric transformations, and the "implicit" one, using generalized dipole moments. The examples shown on the next page were computed by correlating the patterns with their discretely transformed versions. Correlations and geometric transformations were performed by a special incoherent-optical analog-correlator, maximum detection by digital evaluation. The implicit approach, based on comparisons of values at suitably chosen pairs of image points, is favorable for serial processing (Glünder and Kramer 1986) and for the implementation on special purpose parallel computing networks (Glünder 1986).

References

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Samples of the generalized ACFs c_s , c_ψ , $c_{\psi\psi}$ and c_1 of three line-like patterns



Examples of descriptor functions C_s and C_1