Tomogram-Reconstruction by Holographic Methods*

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Besides its historic relevance, this text introduces to the theory of tomographic imaging. It starts with the question about the required number of X-ray photons for certain spatial and grey-level resolutions of the reconstructed sectional image. The systems theoretical analysis of the now outdated "coded source" and "coded aperture" schemes provides general insights to the rôle projection angles play for the depth resolution and it nicely relates to the problem of depth resolution in microscopy. A survey of analog approaches to tomographic imaging reveals the most common principles of image reconstruction from projections. Finally, "spectral tomosynthesis" by coherent-optical computing is proposed for image reconstruction.

Fundamentals of X-ray tomography

Introduction

The aim of X-ray tomography is to reconstruct from projections two- or three-dimensional density functions of an object.

The elementary measurement is the chain product of the absorptions along a ray emitted at one point of the X-ray source and recorded at a point in the detector plane by X-ray film or a scintillation counter. Switching to object densities, i.e. to attenuation coefficients – the logarithms of the absorptions –, then an elementary measurement becomes the corresponding line-integral. The line-integrals of all projections are the coefficients of a system of linear equations that is to be solved for the density function of the object under consideration.

To reconstruct an object essentially means, to recover its densities as differences of nearly equal linear combinations of the measurements.

Noise

The elementary measurements are subject to quantum noise, i.e. one obtains m counts or silver grains with an uncertainty of $\sqrt{m}$. The total number of photons to be recorded or the area of a recording-film depends on the desired quality of

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the reconstructed sectional image. For an image of $n \times n$ pixels and a grey-levels, the required total number N of photons is

$$N = c \cdot n^3 \cdot a^2 ,$$

where n denotes the number of independent grey-values along the diameter of the reconstructed image, $1/a$ the relative error of the reconstructed densities, and c a constant of the order of one (Glünder & Platzer 1978).

If the projections are to be recorded on X-ray film, then the film must be of the size of a developed silver grain times the number of photons given by the above equation. If we consider a sectional image of 1000×1000 pixels and a density resolution of half a Hounsfield unit $(1\,\mathrm{HU} = 1‰)$, then we require in the order of $(10^3)^3 \cdot (2 \cdot 10^3)^2 = 4 \cdot 10^{15}$ [counts]. Assuming silver grains of one micron size, this means about $4 \cdot 10^{15} \cdot 10^{-12} = 4 \cdot 10^3\ [\mathrm{m}^2]$ of recording film for a single tomographic reconstruction.

For $n = 256$ pixels along the image diameter and 5 bit density resolution (32 grey-levels), a conveniently sized piece of X-ray film of about $(13 \times 13)\ \mathrm{cm}^2$ will suffice under the same assumptions. Consequently, there are image qualities for which it is worth looking for methods of analog recording and reconstruction.

Superposition of projections

The reconstruction by linear operations is only feasible if the quantities under consideration are linearly combined. This holds for the established computer aided tomography (CAT), for the recording of so-called “sinograms” (cf. Fig. 2), and for related methods. However, it does not hold for coded source imaging where signals from several elementary measurements commonly overlap at one point in the detector plane, i.e. chain products of absorption values are added before the logarithmic transformation. This is one of the fundamental drawbacks of most coded source imaging techniques.

Depth resolution in coded source imaging

Another shortcoming of coded source imaging and likewise of coded aperture imaging is the incompleteness of the measurements due to practicable source configurations. Consequently, the reconstructed image carries certain interdependencies. Unfortunately, they cannot be corrected for theoretical reasons that will be given in what follows. In spite of that, various attempts have been made to reach depth resolution by coded source imaging. Among others, carefully constructed point arrays are used as X-ray source configurations (Weiss *et al.* 1977). With this approach, one tries to extract distinct object layers by linear filtering of the superposed projections which means a near to point-like system response for the layer under consideration and negligible contributions from the other layers.

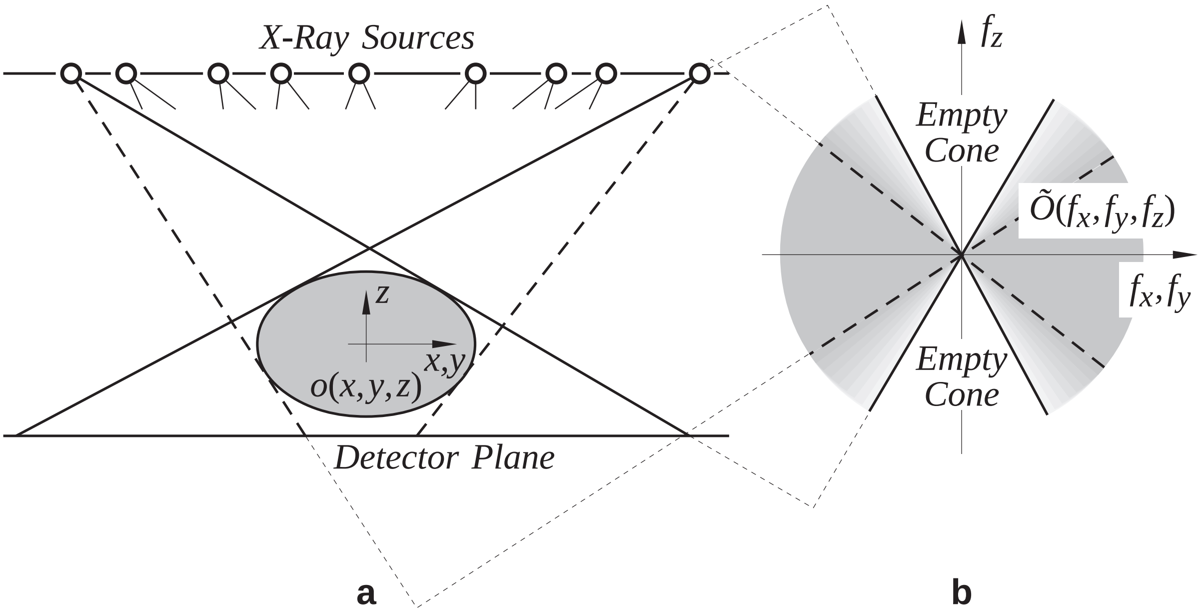

Fig. 1 (**a**) Shadow casting of object $o(x,y,z)$ by several X-ray sources onto a detector plane. (**b**) The Fourier spectrum $\tilde{O}(f_x,f_y,f_z)$ of the object, as far as it is accessible from the shadowgram, is restricted to spatial frequencies outside the "empty cone"

Let us consider an object $o(x,y,z)$ whose x,y-layers are to be reconstructed at various depths z. A typical set-up for coded source imaging is shown in Fig. 1a. The object's Fourier spectrum $\tilde{O}(f_x, f_y, f_z)$, as far as it is accessible from the superposed projections, can be obtained by the "central slice"-theorem (Bracewell 1956) that deals with the Fourier correspondence of a projection (see Sect. 3). To judge the quality of reconstruction, we need to know if and how this representation differs from the ideal object spectrum $O(f_x, f_y, f_z)$. From Fig. 1b the loss of all spectral components in a double-cone around the f_z-axis becomes evident. This "empty cone" results from the projection geometry, i.e. its size is given by the illumination aperture and the object size, and it cannot be avoided with planar coded sources of finite extent. Hence, coded source imaging implies highpass filtering having a corner frequency that linearly rises with depth-resolution. In other words, little to no depth resolution can be achieved for low frequencies f_x, f_y, i.e. the layer crosstalk mainly consists of coarse image structures. Consequently, it is useless to look for algorithms that improve the depth resolution for spatial frequencies f_x, f_y inside the empty cone (Barrett & Chiu 1978).

The use of *a priori* knowledge, for instance that the object density function is non-negative, will not be treated in this paper that is limited to methods of linear filtering and coordinate transformations.

Image reconstruction in analog transaxial tomography

In CAT, three reconstruction methods are used to gain the two-dimensional image of an object slice from one-dimensional projections. Their rôle in analog systems is sketched.

Recursive

Recursive methods are well suited for implementations on digital computers and are not suited for analog techniques.

Deconvolution and back-projection

Presently, this approach is the most widely used technique in CAT. Its implementation on fast computers (pipeline- and array-processors) for convolution operations and coordinate-transformations is attractive.

- Edholm *et al.* (1978) have achieved good reconstructions by a corresponding incoherent optical technique. As with most optical methods, the format of the input data is that of a sinogram.
- Recently, amazing results have been obtained by techniques that combine deconvolution with pointwise reconstructions of object layers (Greivenkamp *et al.* 1978). The drum- as well as the loop-processor incoherent-optically perform the deconvolution and – following opto-electronic conversion – the reconstructed image is synthesized on a CRT-display.
- The commercially available "Simtomix" analog tomograph manufactured by De Oude Delft (Geluk 1976) is based on a similar concept. Deconvolution is also carried out incoherent-optically but image synthesis is performed by a TV-analog processor that limits the attainable image quality.
- Peters (1974) has proposed to coherent-optically filter the so-called "layergram" (back-projection). This technique explicitly operates in the Fourier domain and relates to the following approach.

Spectral synthesis and equalization

"Spectral Tomosynthesis" is essentially based on the "central slice"-theorem. It needs a large number of one-dimensional Fourier transformations and a final two-dimensional Fourier retransformation. Although time-consuming if implemented on a digital computer, coherent-optically computed they appear at the speed of light in the focal-plane of suitable lens systems.

Stroke & Halioua (1977) have proposed a holographic method for three-dimensional reconstructions of specimens in electron-microscopy. It allows one to synthesize in a discrete way the three-dimensional Fourier spectrum of three-dimensional objects from sets of two-dimensional projections. Section 3 presents a similar approach to the reconstruction of sectional images in transaxial tomography, where the image's Fourier spectrum is continuously synthesized.

Other analog methods

Finally, some new methods for analog reconstruction without digital counterpart shall be mentioned that appear highly promising. Nishimura *et al.* (1978) have suggested sequential coherent-optical filtering. Hansen & Goodman (1978) have obtained reconstructions fully in parallel by coherent optical inverse filtering in combination with a spatial weighting and coordinate transformations, for which Hofer (1979) has provided a thorough and much simplified theoretical analysis. Furthermore, he has described in great detail the implementation of the corresponding holographic filter. The method's great advantage is the lack of any mechanic movements, its drawbacks are the modified sinogram – that must be generated in a preceding process – and – if Hofer's technique is used – the inconvenient coordinates of the reconstruction.

Reconstruction by spectral synthesis ("Spectral Tomosynthesis")

The goal of our development was to avoid, as far as possible, complicated mechanic arrangements that are required by the above mentioned methods (except for the last one). Particularly, synchronous mechanic movements of the types listed in Table 1 are difficult to achieve and often are sources of errors. The here proposed concept does not need coupled movements or electronic synchronizations.

Technique	Coupled Movements
backprojection	linear / rotational
drum processor	rotational / rotational / electron-beam position
loop processor	linear / electron-beam position
sequential filtering	linear (x) / linear (y)

According to the "central slice"-theorem, the two-dimensional Fourier transform of a one-dimensional projection of an object only has values along a line through the origin of the spatial-frequency plane that is perpendicular to the direction of the projection (cf. Fig. 1). Hence, if all projections – generated under the continuum of angles $0 \le \varphi < 180^\circ$ – are Fourier transformed and added under the appropriate angles, then the whole spectrum of the object is available and its reconstruction only requires an equalized two-dimensional Fourier retransformation.

The basic idea of the proposed concept is to sequentially select and coherent-optically Fourier transform all projection that are stored as a sinogram (Fig. 2). These one-dimensional Fourier transforms are generally complex-valued and therefore must be recorded as holograms.

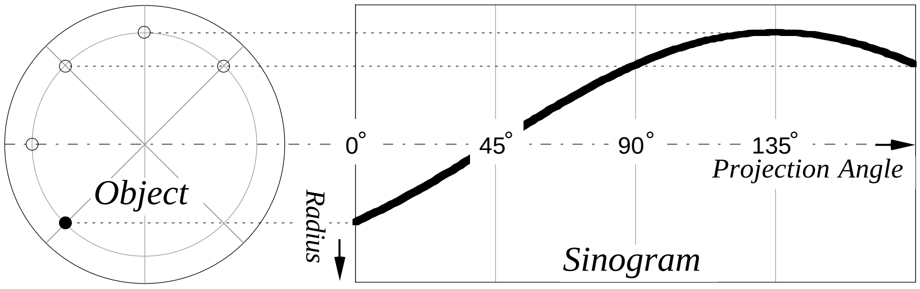

Fig. 2
Conventional sinogram showing the trace of a single object point

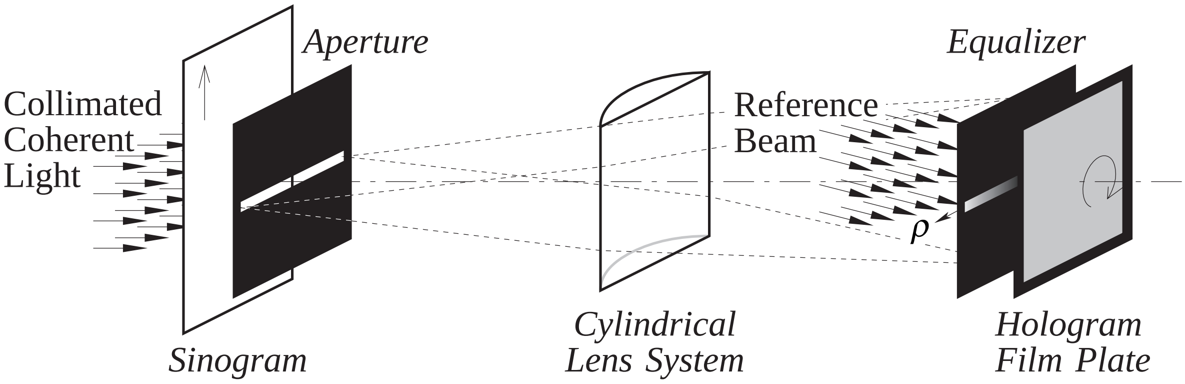

Fig. 3 Principle of the coherent-optically performed spectral tomosynthesis

A coherent-optical processor for this tasks is sketched in Fig. 3. While the whole length of the sinogram passes in front of the slit aperture that sequentially selects the projections, the recording medium rotates through 180° about the optical axis. Every projection is Fourier transformed by the cylindrical lens system and the transforms are successively recorded as holograms. The slit-shaped transparency in front of the recording plane equalizes the spatial frequency spectrum. To compensate for the inevitable $1/f_r$ -loss, the light amplitudes of the interference pattern must be attenuated by $\sqrt{|\rho|}$. A more pronounced attenuation of low spatial frequencies may be desirable with respect to the dynamic range demands of the recording material. They can be reversed during the hologram reconstruction.

Only recently, the principle of holographic spectral synthesis has been experimentally proved at the Optics Lab of the Institute für Nachrichtentechnik of the Technische Universität München.

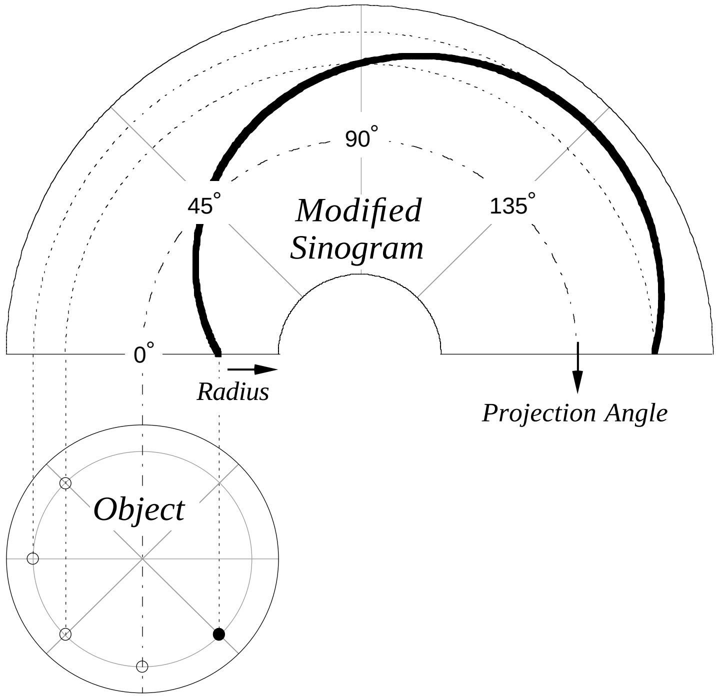

Fig. 4
Modified sinogram showing the trace of a single object point

The remaining problem with the arrangement shown in Fig. 3 is the coupled movement (linear/rotational). It can be solved by a different kind of sinogram. Instead on a rectangular piece of film (Fig. 2), it is recorded on a halfring (Fig. 4), either directly by X-rays, or by geometric transformation of a conventional sinogram, for instance by a cone-mirror.

The final apparatus is depicted in Fig. 5. The modified sinogram is coupled with the film plate. During their 180° rotation, a slit-aperture scans the sinogram. (The slit's shape depends on the sinograms' recording geometry.) Two mirrors bring the signal to the optical axis where it is Fourier transformed by a lens system. A slit-shaped transparency in front of the film plate equalizes the spatial frequency spectrum. The sinograms must be precisely centred, otherwise troublesome phase errors will occur in the recording plane.

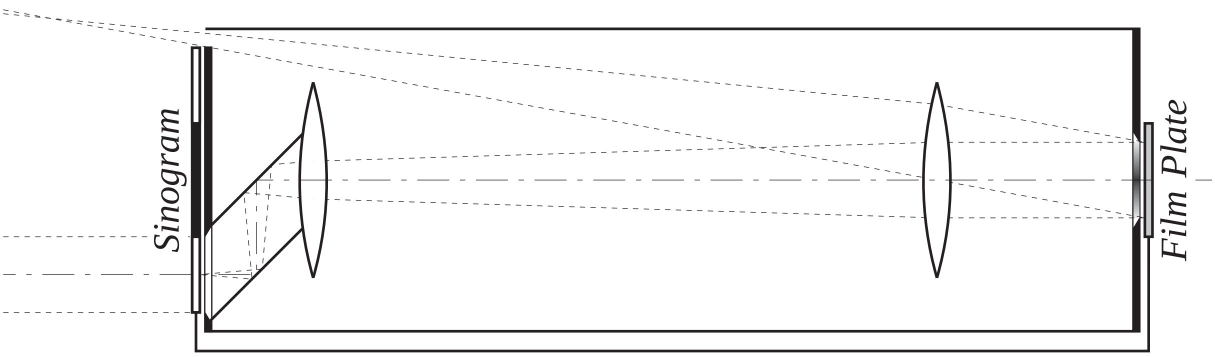

Fig. 5
Coherent-optical processor for spectral tomosynthesis without coupled movements

On-line processing can be achieved with thermoplastic film. Such materials allow the reconstruction of a hologram immediately after its recording.

An experimental processor according to Fig. 5 is being built for tomographic reconstructions from computer generated sinograms.

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References

Barrett H.H. and Chiu M.Y. (1978) Three-dimensional radiographic imaging. In: *Proceedings of the ICO-11 Conference*. Madrid, pp. 135-138.

Bracewell R.N. (1956) Strip integration in radio astronomy. *Australian Journal of Physics* **9**: 198-217.

Edholm P., Hellström L.G. and Jacobson B. (1978) Transverse tomography with incoherent optical reconstruction. *Physics in Medicine and Biology* **23**: 90-99.

Geluk, R.J., c/o De Oude Delft (1976) *Verfahren und Vorrichtungen zum linearen Filtern von zweidimensionalen Signalen.* German Patent: DT 2659427 A1.

Glünder H. and Platzer H. (1978) Bildqualität und Röntgenphotonenzahl bei der Transaxialtomographie. Talk given at the “Unternehmensbereich Medizin der Siemens AG”.

Greivenkamp J.E., Swindell W., Barrett H.H., Gordon S.K. and Gmitro A.F. (1978) Recent advances in uncomputerized transaxial tomography I – The loop-processor. II – The drum-processor. *OSA Annual Meeting*, Papers FE 4 and FE 5.

Hansen E.W. and Goodman J.W. (1978) Optical reconstruction from projections via circular harmonic expansion. *Optics Communication*s **24**: 268-272.

Hofer J. (1979) Optical reconstruction from projections via deconvolution. *Optics Communication*s **29**: 22-26.

Nishimura M., Casasent D. and Caimi F. (1978) Optical inverse Radon transformation. *Optics Communications* **24**: 276-280.

Peters T.M. (1974) Spatial filtering to improve transverse tomography. *IEEE Transactions on Biomedical Engineering* **21**: 214-219.

Stroke G.W. and Halioua M. (1977) Holographic implementation of our 3-D reconstruction method for application in electron microscopy of non crystalline specimens. *Unpublished Report,* private communication.

Weiss H., Klotz E., Linde R., Rabe G. and Tiemens U. (1977) Coded aperture imaging with X-rays (Flashing Tomosynthesis). *Optica Acta* **24**: 305-325.

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