Image quality and X-ray dose in transaxial tomography

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Image quality and X-ray dose in transaxial tomography*

Helmut Glünder and Helmut Platzer

The number of X-ray photons is estimated that is required for the recording of object projections from which a sectional image of desired spatial and density resolution can be computed. We find that $N_{0,\text{total}} = \text{const.} \cdot n^3 \cdot a^2$ *photons must enter the object slice, where n is the number of pixels along the image diameter, a is the number of gray-levels, and the constant is object specific. Of course, the X-ray dose additionally increases with decreasing beam width.*

Transaxial computed tomography (CT)

We derive an estimate of the number of X-ray photons that must be spent for an object's projections in order to achieve a desired spatial and gray-level resolution of a thereof computed sectional image. In general, such tomographic imaging requires $z = \frac{\pi}{2} n$ one-dimensional projections,[1] each of which consists of n equidistant samples according to the Nyquist criterion. We consider projections by a parallel beam that rotates around the object in steps of $\Delta\varphi = 180°/z$. Furthermore, we assume the X-ray source to emit monochromatic radiation and the detectors to have a quantum efficiency of 100% and to collect all photons that leave an object slice.

If an X-ray consisting of N_0 photons enters an object, then $N := N_0/A = N_0 \cdot e^{-p}$ photons can be detected at the opposite side. Consequently, the attenuation coefficient is given by the integral $p = \int \alpha(l)\,\mathrm{d}l$ of the object densities $\alpha(l)$ along the X-ray track. Because we shall consider the reconstruction of spatially discrete densities α_ν by digital computer, it is appropriate to write $p = \sum_i \alpha_i$. If at most n elementary densities contribute to an attenuation coefficient, then n denotes the number of independent density values or pixels along the largest possible object diameter. For our derivation the α_ν are assumed to have similar values with the mean value $\bar{\alpha}$.

* Translation of the slightly reworked manuscript with an added postscript of a lecture titled "Bildqualität und Röntgenphotonenzahl bei der Transaxialtomographie" given at the Unternehmensbereich Medizin of the Siemens AG in Erlangen (7. December 1978). Compiled and edited by the first author and based on the original overhead transparencies and on H. Platzer's lecture notes

1 Unaware of the work of Kowalski and Wagner (1977), we originally assumed the relation $z = n$ but it turns out (cf. equation [4]) that z is canceled in our first order estimation. The actually required number of projections had not been generally accepted until the publication of Herman's classic book in 1980. Around that time, Platzer confirmed this relation and published his derivation in 1981

Quantum noise

For the reconstruction of a sectional image by linear methods, one needs to know the attenuation coefficients $p = \ln(A) = -\ln(N/N_0)$, that is the negative logarithms of the normalized signals from the X-ray detectors that sense the projections. We now propagate the standard deviation $\Delta N = \sqrt{N}$ of a Poisson process (quantum noise) by the transformation $\Delta p \approx \Delta N \left|\partial p/\partial N\right| = 1/\sqrt{N}$ to obtain the relative error of a single attenuation coefficient

$$\frac{\Delta p}{p} = \frac{1}{\sqrt{N}} \cdot \frac{1}{\ln(A)}. \qquad (1)$$

Filtered backprojection

To reconstruct the image of an object slice, all projections must be backprojected under the appropriate angles and then be added to the image array. Either every projection or the resulting layergram (raw image) must be filtered to equalize the $1/f_r$-characteristic. One-dimensional filtering of a projection means to compute the differences of every attenuation coefficient and its weighted neighbors, for example in the crudest form $p_{j,\text{filtered}} = p_j - \frac{1}{2} p_{j-1} - \frac{1}{2} p_{j+1}$.

The superposition of z filtered backprojections reduces the noise of every value in the image array by the factor $1/\sqrt{k \cdot z}$, with $k \geq 1$ expressing the influence of the subtracted neighborhood that involves at least the equivalent of $2z$ values; hence $k \approx 1$ is a reasonable estimate. Consequently, the standard deviation of a typical value $\bar{\alpha} \approx p/n$ in the image array is $\Delta\alpha \approx \Delta p/\sqrt{z}$ and the relative error results as

$$\frac{\Delta\alpha}{\alpha} \approx \frac{n}{\sqrt{z}} \cdot \frac{\Delta p}{p}. \qquad (2)$$

Image quality and photon requirement

Basic measures of image quality are the spatial resolution and the amplitude resolution. In transaxial tomography one generally deals with circular image supports,[2] hence we specify the linear spatial resolution by the n independent densities along the support diameter. The relative error of the values in the image matrix is denoted as density (amplitude) resolution and its inverse $a = \alpha / \Delta\alpha$ can be regarded as the number of discernible gray-levels that signify the X-ray densities in an object slice.

By inserting equation (1) to (2) and with $z = \frac{\pi}{2} n$, we obtain – for a desired image quality – an estimate of the required X-ray photons in a single detector

$$N \approx \frac{n^2 \cdot a^2}{z} \frac{1}{\left[\ln(A)\right]^2} = \frac{2}{\pi} \frac{n \cdot a^2}{\left[\ln(A)\right]^2} . \qquad (3)$$

Because of the n detectors and the z projections, the total number of photons[3] that need to be detected per sectional image becomes

$$N_{\text{total}} \approx \frac{n^3 \cdot a^2}{\left[\ln(A_{\max})\right]^2} , \qquad (4)$$

where $A_{\max}$ is the maximum X-ray attenuation of an object slice.

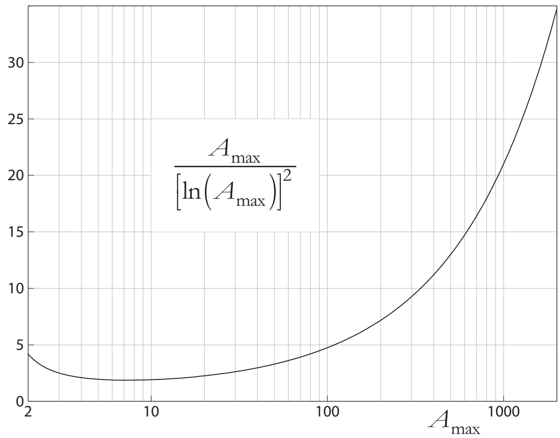
X-ray dose

To estimate the X-ray dose, we need to know the total number of photons that enter the object. With definition $N_0 = A \cdot N$ we get $N_{0,\text{total}} = A_{\max} \cdot N_{\text{total}}$ and the total number of absorbed photons becomes $N_{A,\text{total}} = N_{0,\text{total}}\left(1 - 1/A_{\max}\right)$. The maximum attenuation of body or head slices commonly is $A_{\max} > 10$ which suggests the approximation

$$N_{A,\text{total}} \approx N_{0,\text{total}} \approx n^3 \cdot a^2 \frac{A_{\max}}{\left[\ln(A_{\max})\right]^2} . \qquad (5)$$

In addition to these considerations, absolute dimensions need to be considered, namely the illuminated area or volume. For $N_{0,\text{total}} = \text{const.}$, the X-ray dose linearly increases with decreasing beam width, that is with increasing depth resolution.

2 Platzer (1985) showed that differently shaped supports can be economically scanned by appropriately chosen uneven angle increments
3 Most remarkable this estimate does not dependent on z



Object specific weighting term as a function of the maximum X-ray attenuation of an object slice for values relevant in medical diagnostics

Conventional X-ray shadowgram

We consider conventional X-ray shadowgrams of $n \times n$ pixels with an amplitude resolution $1/a = \Delta p / p$. By isolation of N in equation (1) and by taking into account the n^2 attenuation coefficients, the required number of photons in the detector plane becomes

$$\tilde{N}_{\text{total}} \approx \frac{n^2 \cdot a^2}{\left[\ln(A_{\max})\right]^2} . \qquad (6)$$

Hence, $\tilde{N}_{0,\text{total}} = A_{\max} \cdot \tilde{N}_{\text{total}}$ photons must enter the considered square region of the object.

Historical Example[4]

We estimate the absorbed dose and the exposure for a single CT-scan of a head phantom. The sectional image is assumed to consist of 80×80 pixels with a density resolution of 0.5%. Along the object's maximum diameter $l_{\max} = 180\,\text{mm}$ the phantom presents 16 mm of dense bone of $\mu_{b,70\,\text{keV}} = 0.05/\text{mm}$ as well as brain tissue of $\mu_{t,70\,\text{keV}} = 0.02/\text{mm}$ for the remainder.[5] Accordingly, the maximum attenuation is $A_{\max} \approx 59.1$ and $N_{0,\text{total}} \approx 7.28 \cdot 10^{10}$ X-ray photons are required.

4 Differently from our original text, a scan is considered, similar to that performed by the EMI CT head scanner which used 160 detector positions but produced slice images of only 80×80 pixels with a density resolution of 0.5% by non-linear reconstruction (Hounsfield 1973)
5 Values taken from the "ICRU Report 44" (1989) for $E = 70\,\text{keV}$

For this amount of absorbed photons of the effective energy $E = 70\,\mathrm{keV} \approx 1.1 \cdot 10^{-14}\,\mathrm{J}$ and the mass $m \approx 0.57\,\mathrm{kg}$ of a slice as thick as the effective beam width $d = 20\,\mathrm{mm}$, we obtain the absorbed dose $D \approx N_{0,\mathrm{total}} \cdot E/m \approx 1.4\,\mathrm{mGy} = 0.14\,\mathrm{rd}$. The scan-equivalent photon density in the X-ray beam is $Z = N_{0,\mathrm{total}} / \left(d \cdot l_{\max}\right) \approx 2 \cdot 10^7/\mathrm{mm}^2$ which, with $c_{\mathrm{air},70\,\mathrm{keV}} \approx 5.8 \cdot 10^{-12}\,\mathrm{mm}^2\,\mathrm{C/kg}$,[6] results in the exposure of $J = c_{\mathrm{air},70\,\mathrm{keV}} \cdot Z \approx 0.12\,\mathrm{mC/kg} = 0.45\,\mathrm{R}$.[7]

Under comparable conditions, a conventional X-ray shadowgram of the same head phantom that shows 80×80 pixels and 200 discernible gray-levels – which is generally unsuited for medical diagnostics – requires $\tilde{N}_{0,\mathrm{total}} = N_{0,\mathrm{total}}/n \approx 9.1 \cdot 10^8$ photons. With $\tilde{m} \approx 4.3\,\mathrm{kg}$, the absorbed dose becomes $\tilde{D} \approx \tilde{N}_{0,\mathrm{total}} \cdot E/\tilde{m} \approx 2.4\,\mu\mathrm{Gy} = 0.24\,\mathrm{mrd}$ and, with the photon density $\tilde{Z} = \tilde{N}_{0,\mathrm{total}} / l_{\max}^2 \approx 2.8 \cdot 10^4/\mathrm{mm}^2$ in the parallel X-ray illumination, the exposure results in $\tilde{J} = c_{\mathrm{air},70\,\mathrm{keV}} \cdot \tilde{Z} \approx 0.16\,\mu\mathrm{C/kg} = 0.63\,\mathrm{mR}$.

Postscript (2011)

At least during the first decade of CT-use, published data about the X-ray doses was scarce. However, our results did not even enter the otherwise rather detailed book about the foundations and the technology of imaging systems for medical diagnostics (Krestel 1980) that was issued by the Siemens AG two years after our lecture; and it seems worth mentioning that we did not receive any comments in response to our presentation. In recent years, the situation has much improved through extensive investigations, such as the NEXT-survey (2007) and corresponding recommendations, such as the EC-guidelines (2001/2008).

Over the years, CT radiation dose has in fact been lowered. However, if a reduction results from a higher quantum efficiency of the X-ray detectors, we should like to point out that our dose estimation already assumes a quantum efficiency of 100%. Though there are other measures, such as improved beam collimation, some techniques require more careful evaluations especially regarding their impact on image quality; among them spiral/helical scans. Despite all efforts to reduce CT radiation dose, it is important to realize that they are limited by physics. Consequently, as derived here and now confirmed by several published experimental studies, a diagnostically relevant CT-scan of the human head or body definitely means *at least* an absorbed dose of the order of what the US Environmental Protection Agency (EPA) regards as the yearly limit for ionizing radiation beyond the natural background dose, namely 1mSv. More illustratively, the EC-guidelines equate the radiation dose of a typical abdominal CT-scan to that of about 500 conventional X-ray shadowgrams of the chest.

With the availability of Magnetic Resonance Imaging (MRI) and of advanced ultrasound imaging – techniques that do not use ionizing radiation – the question may arise, why is it that CT is still widely used? With respect to MRI, the most important reason is lower costs of the apparatus and of its operation, and hence its much wider distribution. The latter is especially important when urgent decisions for treatments require immediate diagnoses, as with cerebrovascular accidents (stroke), for which ultrasound is unsuited. Interestingly, diagnostic brain imaging was an essential motive for and during the development of CT.

6 This constant is defined as $c = \dfrac{\mathrm{e} \cdot \mu(E)}{w \cdot \rho} E$, with e: elementary charge, μ: absorption coefficient, w: ionization energy, and ρ: density

7 For somewhat different spatial and density resolutions McCullough *et al.* (1973) have measured exposures in the range of $0.6 \ldots 1.5\,\mathrm{R}$ in a head phantom when scanned by the EMI-Scanner. (The report was kindly provided to us by G.W. Stroke in March 1978)
Hounsfield (1973) reports a total exposure of $1.9\,\mathrm{R}$ for six scans that covered the whole head ($d \approx 25\,\mathrm{mm}$) and that resulted in slice images of $n = 80$ and $a = 200$

References

Experts representing European radiology and nuclear medicine and UK Royal College of Radiologists (2001/2008) *Referral guidelines for imaging*. Radiation Protection Booklet No. 118, Office for Official Publications of the European Communities, Luxembourg.

Herman G.T. (1980) *Image reconstruction from projections*. Academic Press, New York.

ICRU (1989) *Tissue substitutes in radiation dosimetry and measurement*. Report No. 44, International Commission on Radiation Units and Measurements, Bethesda/MD.

Kowalski G. and Wagner W. (1977) Generation of pictures by X-ray scanners. *Optica Acta* **24**: 327-348.

Krestel E. (ed.) (1980) *Bildgebende Systeme für die medizinische Diagnostik. Grundlagen, Technik, Bildgüte*. Siemens AG, Berlin.

McCullough E.C., Baker H.L. Jr., Houser O.W. and Reese D.F. (1973) *An evaluation of the quantitative and radiation features of a scanning X-ray transverse axial tomograph (EMI-Scanner)*. Report, Mayo Clinic, Department of Diagnostic Roentgenology, Rochester/MN.

Platzer H. (1981) Optical image processing. In: Oja E. and Simula O. (eds.) *The second Scandinavian conference on image analysis*. Pattern Recognition Society of Finland, Espoo/Finland, pp. 128-138.

Platzer H. (1985) Abtastung durch winkelperiodische Geradenbüschel: Das Sampling-Theorem der Computertomographie. In: Binkert G. (ed.) *Festschrift zum 60. Geburtstag von Prof. Dr.-Ing. Hans Marko*. Lehrstuhl für Nachrichtentechnik der Technischen Universität, München, pp. 109-115.

Stern S.H., CRCPD and American College of Radiology (eds.) (2007) *Nationwide evaluation of X-ray trends (NEXT)*. Tabulation and Graphical Summary of 2000 Survey of Computed Tomography, Conference of Radiation Control Program Directors Inc., Frankfort/KY.